1. Fundamental and derivate quantities and units

- Fundamental Physical quantities are those that can be directly defined, and for which the units are chosen arbitrarily, independent of others physical quantities.

No.	Fundamental Quantity	Unit	Symbol	Dimension
1	Length	Meter	m	L
2	Time	Second	s	T
3	Mass	Kilogram	kg	M
4	Current intensity	Ampere	A	I
5	Light intensity	Candela	Cd	J
6	Temperature	Kelvin	K	θ
7	Solid angle	Ste-radian	Sr	
8	Plane angle	Radian	rad	

- The derivative physical quantities are those that are defined indirectly. They have the measurement units' functions of fundamental units.

If we have:

- $U = U(L, T, M, I, J, \theta)$
- $[U] = L^{\alpha}T^{\beta}M^{\gamma}I^{\delta}J^{\varepsilon}\theta^{\xi}$

2 The homogeneity of physical equations:

- The physical formulas are invariant to the measurement units' transformation.

3 **Examples**:

3.1 Velocity:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad [v] = \frac{[x]}{[t]} = \frac{m}{s} = m \cdot s^{-1} = L \cdot T^{-1} \tag{1}$$

3.2 Acceleration:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad [a] = \frac{[v]}{[t]} = \frac{m}{s^2} = m \cdot s^{-2} = L \cdot T^{-2} \tag{2}$$

3.3 Impulse:

$$p = m \cdot v \qquad [p] = [m] \cdot [v] = kg \frac{m}{s} = kg \cdot m \cdot s^{-1} = M \cdot L \cdot T^{-1} \tag{3}$$

3.4 Force:

$$F \underset{\Delta t \to 0}{\lim} = \frac{\Delta p}{\Delta t} = \frac{dp}{dt} = \frac{dmv}{dt} = m\frac{dv}{dt} = m \cdot a$$

$$[F] = \frac{[p]}{[t]} = \frac{kg \cdot m}{s^2} = kg \cdot m \cdot s^{-2} = M \cdot L \cdot T^{-2} \qquad [F] = 1\frac{kg \cdot m}{s^2} = 1N \qquad (4)$$

3.5 Pressure:

$$P = \frac{dF}{dS} \qquad [P] = \frac{[F]}{[S]} = \frac{kg}{s^2 \cdot m} = kg \cdot m^{-1} \cdot s^{-2} = M \cdot L^{-1} \cdot T^{-2} \qquad [P] = 1\frac{N}{m^2} = 1Pa \qquad (5)$$

3.6 Mechanical work:

$$dL = F \cdot ds \qquad [L] = [F] \cdot [s] = kg\frac{m^2}{s^2} = kg \cdot m^2 \cdot s^{-2} = M \cdot L^2 \cdot T^{-2} \qquad [L] = 1N \cdot m = 1J \quad (6)$$

3.7 Kinetic energy:

$$E_c = \frac{m \cdot v^2}{2};$$

$$[E_c] = [m][v]^2 = \frac{kg \cdot m^2}{s^2} = kg \cdot m^2 \cdot s^{-2} = M \cdot L^2 \cdot T^{-2}; \quad [E_c] = 1kg \cdot m^2 \cdot s^{-2} = 1J \quad (7)$$

3.8 Gravitational potential Energy:

$$E_p = m \cdot g \cdot h$$

$$[E_p] = [m] \cdot [g] \cdot [h] = kg\frac{m^2}{s^2} = kg \cdot m^2 \cdot s^{-2} = M \cdot L^2 \cdot T^{-2}; \; [E_p] = 1kg \cdot m^2 \cdot s^{-2} = 1J \quad (8)$$

3.9 Elastically potential Energy:

$$E_{p,e} = \frac{k \cdot x^2}{2}$$

$$[E_{p,e}] = [k][x]^2 = \frac{kg \cdot m^2}{s^2} = kg \cdot m^2 \cdot s^{-2} = M \cdot L^2 \cdot T^{-2}; \; [E_{p,e}] = 1kg \cdot m^2 \cdot s^{-2} = 1J \quad (9)$$

3.10 Power:

$$P = \frac{dE}{dt}$$

$$[P] = \frac{[E]}{[t]} = \frac{kg \cdot m^2}{s^3} = kg \cdot m^2 \cdot s^{-3} = M \cdot L^2 \cdot T^{-3} \qquad [P] = 1\frac{J}{s} = 1W \qquad (10)$$

3.11 Density:

$$\rho = \frac{dm}{dV} \qquad [\rho] = \frac{[m]}{[V]} = \frac{kg}{m^3} = kg \cdot m^{-3} = M \cdot L^{-3} \qquad (11)$$

3.12 Angular velocity:

$$\omega = \frac{d\varphi}{dt} \qquad [\omega] = \frac{[\varphi]}{[t]} = \frac{rad}{s} = rad \cdot s^{-1} = T^{-1} \qquad (12)$$

3.13 Solid angle:

$$d\Omega = \frac{dA}{R^2} \qquad [\Omega] = \frac{[A]}{[R]^2} = \frac{m^2}{m^2} = L^0 = 1 \qquad [\Omega] = 1L^0 = 1 steradian \qquad (13)$$

4 Establishes into a constant approximation the formula of a gravitational (mathematical) pendulum, T using the homogeneity of the dimensional equations.

5 The mass of a parachute with jumper (parachutist) is m = 100 kg, and is launched from a tower completely open with no initial velocity. Find the velocity expression, v(t) and the velocity limit if we know that the resistance force is proportional with the velocity, R = kv, where k = 500 Ns/m.

6. A skier with the weight G descends a hill that make an angle α with the horizontal plane. The motion equations are $x = Agt^2$ along the hill land y = 0, where g is the gravitational acceleration and ***A*** a constant coefficient. How much is the friction force between skier and snow on the hill. (Particular case: the skier mass m = 70 kg, angle $\alpha = 30^0$; A = 0.1).

7. A material point with mass *m* is moving along a trajectory given by the Cartesian components: $x(t) = A\cos(kt)$ and $y(t) = B\cos(kt)$. Characterize the force F that produces this type of motion if we know that the force depends only by the material point position. Give some examples.

8. On a body with a mass ***m*** = 2 kg are acting two forces, $\boldsymbol{F}_1$ = 3 N and $\boldsymbol{F}_2$ = 4 N, which are characterized by the angles $\alpha_1 = 60^0$, and $\alpha_2 = 120^0$ respectively, with the direction of velocity $\vec{v}_0$. Find the body acceleration, ***a***, velocity, **v** and the distance covered into a time t = 10 s starting from the beginning of motion (Particular case v_0 = 20 m/s).

9. A tractor is traveling with a velocity of v_0 = 36 km/h. If the radius of the wheel is R = 0.5 m find out:

a) The parametric motion equations of a point from the external wheel circumference.

b) The tangential velocity components and the value of velocity.

c) The path distance by a point between two contacts with the road.

10. A body with a mass m = 5 kg can slide with friction on a horizontal surface if is pushed by a spiral spring, with an elastic constant k = 200 N/m, and which was compressed at half of his length l = 20 cm. Find out:

a) the mechanical work of friction force during expansion.

b) how much must is the friction coefficient if in the final position the spring in not stressed.

11. A concrete cube with the side of a = 0.8 m and a density of 2500kg/m^3 must be tipped around an edge. Calculate:

a) Point of application direction and value of the minimal force needed to turn turn-over;

b) The expression of a horizontal force that can turn-over the concrete block if this is applied on superior edge as function of rotation angle.

c) The mechanical work spent for turning over the concrete block.

12. On an inclined plane with inclination α = arcsin(3/5) and length l = 2.1 m is rolling with no sliding friction an homogeneous sphere of mass m = 40 g and radius R = 10 mm. How much is the transversal velocity, angular velocity and frequency at the base of the plane.

13. Demonstrate that after a perfect elastic collision of two hokey pucks of the same mass initial one being at rest the angle between the directions of pucks is 90^0. How much are the velocities and scatter angle θ_2 if the initial velocity of projectile puck is v = 40 km/h and $\theta_1 = 60^0$?